MEASURING BOND PORTFOLIO VALUE AT RISK- US AND TAIWAN GOVERNMENT BOND MARKETS EMPIRICAL RESEARCH

BY

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Abstract

This paper concerns with the yield curve modeling, factor sensitivities, VaR and VaR component measurement of the US Treasury Bond portfolio. Like the major point of stock portfolios on variance covariance matrix, the bond portfolios are on the yield curve modeling. To model the yield curve, RiskMetrics has used the key rate yield curve modeling. Though it is intuition and applicable, the multifactor model would bring in many parameter estimation problems such as inflation of parameter covariance and difficulty of distinguishing components of risk factor, which leads to confusion for bond managers. Alternatively, Nelson and Siegel (1987) have proposed the parsimonious yield curve factor reduced modeling- level, slope, and curvature (LSC), later extended by Wilmer (1996). The three factors of the LSC yield curve model represent different risk component and are easy to distinguish among them but they still have some extent of correlation. Further yield curve model is the principle component factor based model. Principle component can be independent of each other and using only a few factors can explain the most of bond price change through the decomposition of the yield change covariance, especially for the short term dynamic yield change. For the measurement of bond VaR, after yield curve modeling, we can apply the factor sensitivities such as the key rate duration, level, slope, curvature duration, and principal component duration, and the factor covariance to calibrate the bond's VaR figure. In final, we offer new tool: structural equation modeling (SEM) for the interest rate term structure research and its VaR measures.

Keywords: Value at Risk, Key rate Duration, Level Slope Curve, Principal Component, and SEM.
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CHAPTER I
RESEARCH BACKGROUND AND OBJECTIVES

1. Research Background

The bond value is subject to the yield curve movement. Many researchers such as Bloomberg, Standard & Poor, Morgan Stanley, and JP Morgan have the US yield curve chart movement displayed every day on their web sites. We can observe from there how the yield curve flows with the US economic change. Notably, the yield curve flows dramatically when the federal reserve bank announces the interest rate change. Typically, the yield curve will be upflow when the US economic is strong and downflow when the US economic is weak. While the yield curve movement is the indicator of fixed income portfolio value change, to measure and explain fixed income portfolio value change is equal to analyze the yield curve structure and its momentum. We often use simple tool like the bond duration to describe the bond price sensitivity with respect to the interest rate change. Anyhow, the major flaw of duration that assumes flat flow yield curve movement makes it hard to match the real world yield curve movement. In reality, the yield curve would be up flow or down flow change in addition to the rarely flat flow change. Therefore, we should find some ways to describe the whole yield curve behavior and analyze and detail the yield curve structure. The bootstrapping is some simple way to model the yield curve. It uses the some bond yield and searches the full yield curve structure. The others are the functional yield curve model derived by many yield curve researchers such as Mcculloch [1975], Vasicek and Fong [1982]. Macculloch first used the cubic spline function to model the yield curve function and later Vasicek and Fong developed the cubic spline function into the exponential spline yield curve function. Those yield curve modelings can be used further to measure the bond value changes with respect to
the yield model factor changes. For instance, the one year zero bond value change can be measured by simply finding the one yield factor duration and the different maturity zero bond value change can be measured by the related yield factor duration. The spline yield curve modeling also can be used to find the yield sensitivity to the factor changes. In other words, the bond would be viewed as the different cash flow combination and we should try to decompose those cash flows into our yield curve modeling factor “blocks”, and then find the price sensitivity of those blocks, and then the VaR components and the VaR value.

In practice, JP Morgan’s RiskMetrics in its 1995 technical document has proposed the key rate duration to measure the bond portfolio risk, VaR component, and VaR value. The key rates act as the major yield factors and thus we can easily find the variance covariance of those key rate “factor blocks” to find the bond yield risk. Golub [1997] has derived the key rate “factor block” duration that can be used for the VaR measure and VaR components of portfolio bond. Many bond brokers have found this bond risk measure is easy to apply since the key rate factor can be understood and defined specifically. However, this key rate risk measure should forecast the short term variance and covariance yield matrix to implement the bond risk measure. Otherwise, the key rate risk measure will be only fit to the long term bond risk measure.

As to the functional yield modeling, the parsimonious three factor-level slope curvature-yield curve modeling proposed by Nelson and Siegel [1987] and Jones [1991] will be more valuable than the other functional yield curve modeling mentioned foregoing. The three factors have been functioned to become sensible to the economic factor behaviors such as the inflation, business cycle, and yield volatility. Specifically, the functional yield curve level factor would be subject to the inflation change, the functional yield curve slope factor would be subject to the business cycle change, and the functional yield curve curvature factor would be subject to the interest rate
volatility. Thanks to the three factor property, the bond risk measure could be described and explained by the real world economic factors- inflation, business cycle, and yield volatility. Further, Willner [1997] continued the three factor model and developed the three factor durations (cash flow weighted factor sensitivity) to deal with the bond portfolio risk measure and VaR measure. Moreover, the three factor yield curve modeling has the advantage of parsimonious and can reduce the application error of redundant factor model such as the key rate yield curve model estimation mistake.

Another way to model the yield curve is the factor variance and covariance matrix transformation named principle component analysis (PCA). Litterman and Scheinkman [1991] applied it for yield curve modeling and Willner [1996] developed the principal components durations for the bond portfolio risk measure. PCA tried to identify the fewest number of new explanatory variables to account for the variability of the yield curve change. But, unlike the functional three factor yield curve approach, the principal components would not be allowed any relationship among factor components. For investors, this might eliminate the factor variance covariance matrix consideration while the key rate or the three factor yield curve modeling would do so. However, the principal components couldn’t be easy to identify and define as the real world bond risk factors.

The final approach to model the yield curve would be the application of structural equation modeling that encompasses many research subjects such as the covariance structure analysis, latent variable analysis, and exploratory factor analysis. Moreover, with the help of LISREL and Amos, SEM has been applied in more respects (see e.g. Austin & Calderon [1996], Tremblay & Gardner [1996]). We will adopt the SEM exploratory factor analysis that will search for the parsimonious unobserved latent factors to well explain the observed known endogenous variable’s variance.
Unlike the PCA that requires the factor components’ zero relationship (orthogonal), the SEM exploratory factor will allow some kind of relationship among latent factors. The approach is similar to the three factor model but is more flexible i.e. not restricted to the level slope curvature factor function definition, and, therefore like PCA, the SEM exploratory factor model wouldn’t be easy to be defined and identified for practice investors’ use.

2. Research Objective

We will apply the four models described foregoing to analyze and detail the yield curve movement for the US and Taiwan government bond markets. The US bond market currently measures over 1,000 government securities, over 10,000 corporate bonds, and well over 700,000 mortgage-backed securities. The total market value estimates in excess of four trillion US dollars. For the study of the US bond market, the treasury yield curve behavior becomes the focus subject of many institutions and investors and the market causality suggests the US bond market yield curve behavior also would influence the global bond markets. On the other side, in 1995 Taiwan launched the plan to be an Asian financial center and began to open wider its capital markets. By 1997, Taiwan government bond issues had grown to about one trillion NT$ and most been traded by domestic institutions. The foreign investors still play with the local investors on the ground of stock market. In the near future, Taiwan will be the WTO member, and we expect that the Taiwan government bond market will be traded actively and liberally. Thus, certainly the yield curve study of Taiwan bond market is important and necessary for the growing Taiwan bond market risk measure. To compare between US and Taiwan bond market risks with respect to yield curve model factors or economic factors and to search the two bond market relationships will become interesting and valuable.
Since yield curve construction has its own advantage and disadvantage, in reality we will implement the model risk measure of US bond market and Taiwan bond market and also fulfill the comparisons of both bond market risks. The bullet and the barbell bond portfolio will be the study cases for comparison between both bond market’s risks. The weighting of Riskmetrics bond index can also be used for both market’s comparison. Finally, we also could construct the bond portfolio frontier for both bond markets by means of the bond risk measure as completing the foregoing four yield curve modelings.
CHAPTER II
LITERATURE REVIEW

With respect to the interest rate risk and value at risk empirical study, J P Morgan in its 1995 year "technical document" has detail description for the key rate variance-covariance value at risk measurement. In reality, key rates play the important role for the standardized risk factors. As to the individual bond or bond portfolio, we can use just J P Morgan provided key rate parameters (variance/ covariance matrix and key rates), and obtain the bond value at risk easily. If the bond cash flow periods don't match the key rate periods used in RiskMetrics, then by the cash flow distribution methods, e.g. duration mapping or variance equality, we would find way to distribute the unmatched bond cash flow to the key rate vertices, i.e. the standardized key rates. As the same method, we can use the key rate parameters provided by J P Morgan to acquire the linear value at risk. In addition, key rate duration (Ho[1992]) also can be applied to the cash flow mapping and bond value at risk measure. Golub[1997] has applied this method for the derivation of bond value at risk formulation.

RiskMetrics or key rate duration both retain the delinquency of dependence between different key rates. In other words, how the relationships of key rate factors catch the real word yield curve movement is the key point for the key rate application of bond value at risk. However, the key rate factor dependence will become obstacle for investor uses since they must consider how well the key rate factor relationships match the real thing yield curve movement.

In order to catch the yield curve movement, we can find many mathematic interpolation theory applied for the term structure of yield curve moment and further for the bond value at risk analysis. For example, Mcculloch [1975] first presented cubic spline discount function to model the yield curve term structure. Later, Vasicek and Fong[1982] improved and extended this function to
the exponential spline discount function that would become suitable for the real world yield curve movement model. Since 1990, we could find many researchers provided plausible yield curve model based on the exponential spline discount function. For instance, Nelson and Siegel[1987] and Jones[1991] offered three factor yield curve model: level, slope, and curvature. This yield curve model is a linear model consisted of level, slope, and curvature factors. If we can pick up or optimize the reasonable interest rate hump point $\tau$, and estimate level, slope, and curvature factor parameters, then it will be a good method of using three factors to explain the yield curve movement due to the economic condition change. Willner [1997] succeeded the three factor yield curve model, and discussed the three factor relation to the bond duration. Thus in his article we can find level duration, slope duration, and curvature duration definition and calculation. Typically, the three factor durations have some kind of relation to the key rate durations. We recall that we can estimate the key rate relationships, and their key rate durations to measure the bond value at risk. Similarly, we can estimate the relationships of three factors and their individual durations to measure the bond value at risk as well.

Another linear method for the bond value at risk measure is the principle component analysis. Principle component analysis can extract the much more yield change components to the major few components like three factor model and thus reduce the number of parameter estimates for the measure of the bond value at risk. The major few component variances can explain almost 99% system variance and most importantly the components will be independent between each other. Singh (1997) compared the performances of the key rate duration, three factor yield curve model, and offered the principal component analysis of the yield curve risk. Golub (1997) further presented the principal component durations to measure the bond value at risk.
CHAPTER III
RESEARCH METHODOLOGY

There will be four linear models used for our bond VaR measure: key rate duration, level slope curvature, principle component analysis, and structure equation model (SEM) as the last one. After modeling the factor’s coefficients, we can calibrate VaR and factor component VaR through the factor’s coefficient transformation to risk measure. As we find all the model parameters, we also will compare each model description of the yield curve behavior. While each model will explain the yield curve in different respect, we will find the advantage and disadvantage of each yield curve modeling from different prospect such as the model factor representations, sensitivities, correlation, variance covariance, and the overall model significant tests.

1. Bond Portfolio VaR Measure Model I: Key Rate Duration Structure and its VaR

The construction of key rate duration can be referred to RiskMetrics technology document. First, it says the yield curve would be divided into several knots or vertex according to the preselected maturity months or years named key rate. Then, the actual cash flow of different maturity bond will be distributed into the preselected key rate. There are two methods to distribute the actual cash flow into the preselected key rate. One is the duration mapping, and the other is the cash flow mapping. We will explain later.

While $c_i$ is the distributed key rate cash flow, $r_i$ is the $i$th key rate ($i=1,2,...,n$), and $V$ is the total portfolio position, the key rate duration can be defined as the followings:

$$k(i)= -(dvi/dr_i)/V$$

$$= ci/[(1+r1)...(1+ri)]*MDuri /V$$

$$= vi/V*MDuri,$$  \hspace{1cm} (1)
where \( M_{duri} \) is the \( i \)th modified duration, \( v_i \) is the \( i \)th distributed key rate cash flow. As to the VaR of bond portfolio, we can refer to the following portfolio VaR function (within 5% confidence level)

\[
VaR = 1.65 \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} v_i \sigma_i^p v_j \sigma_j^p \text{corr}(i,j)},
\]

(2)

where \( v_i \) and \( v_j \) are the \( i \)th and \( j \)th key rate cash flow position, and \( \sigma_i^p \), \( \sigma_j^p \), and \( \text{corr}(i,j) \) are the \( i \)th and \( j \)th key rate cash flow variance and correlation. And, the first derivative of the bond price function (proposed by Fisher [1966]) is

\[
dP = -M_{dur} P dy.
\]

(3)

Taking the variance of bond side equation, we obtain

\[
\sigma^P = M_{Dur y} \sigma^y.
\]

(4)

And, the VaR is

\[
VaR = 1.65 V \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} k_i y_i \sigma_j^y k_j y_j \sigma_j^y \text{corr}(i,j)},
\]

(5)

\[
VaR = 1.65 V \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} k_i k_j \text{cov}(\Delta y_i, \Delta y_j)}.
\]

(6)

We know \( \frac{\Delta V}{V} = -\sum_{i=1}^{n} k_i \Delta y_i \) and if we rewrite the above equation in terms of matrix expression, we can find

\[
VaR = 1.65 V \sqrt{k \Omega k'},
\]

(7)

where \( \Omega \) is the variance covariance matrix of the yield change.

The cash flow mapping is adopted by RiskMetrics. Under the principle of volatility equality, the actual cash flows are distributed into the preselected key rates. The procedure is to find the
parameters of $\alpha$, and $w$ that keep the volatility equal after mapping as the following equations shown.

\[
\sigma = \alpha \sigma_5 + (1 - \alpha) \sigma_7 \quad 0 \leq \alpha \leq 1 \tag{8}
\]

\[
\sigma^2 = w_i^2 \sigma_i^2 + 2 w_i (1 - w_i) \rho_{i,j} \sigma_i \sigma_j + (1 - w_i)^2 \sigma_j^2, \tag{9}
\]

where $\alpha$ is the maturity weight of volatility and $w$ is the weight of cash flow mapping. If we can solve the two parameters $\alpha$ and $w$, then we can use above key rate duration to measure VaR.

2. Bond Portfolio VaR Measure Model II: Level Slope Curvature Yield Curve Modeling and its VaR

The level slope curvature yield curve modeling was derived by Nelson and Siegel[1987]. The yield curve model is used to explain the effects of bond yield curve change due to the economic factor changes such as inflation, business cycle, and interest rate volatility changes. We derive the model as follows:

\[
\tau - \tau - \tau + + = \frac{\tau}{m} \frac{C_e}{m} \tau, \tag{10}
\]

and define $X_1(m) = (1 - X_2(m))/(-m/\tau)$ and $X_2(m) = e^{-mv/\tau}$; thus the yield curve function becomes

\[
Y(L, S, C, m) = L + (L + C) \left(1 - \frac{e^{-m/\tau}}{m/\tau}\right) - Ce^{-m\tau}, \tag{11}
\]

To acquire the level slope curvature durations, we perform the derivation of the yield curve function and obtain

\[
\frac{dY}{dL} = \frac{dY}{dL} \Delta L + \frac{dY}{dS} \Delta S + \frac{dY}{dC} \Delta C \tag{12}
\]

\[
\frac{dY}{dL} = 1, \quad \frac{dY}{dS} = X_1(m), \quad \frac{dY}{dC} = (X_1(m) - X_2(m)). \tag{13}
\]

If the bond value is
\[ B(m) = \sum \frac{c_t}{(1 + Y(m))^t}, \]  

(c\(_t\) is the t\(^{th}\) bond cash flow), further the bond price changes can be described as

\[
\frac{dB}{dY} = \frac{dB}{dY} \left( \frac{dY}{dL} \Delta L + \frac{dY}{dS} \Delta S + \frac{dY}{dC} \Delta C \right)
\]

\[
= \frac{dB}{dY} \left( dL + XI(m)dS + (XI(m) - X2(m))dC \right). \quad (15)
\]

And, through the known three factor coefficients, we can derive the VaR as the following equation shown:

\[
\text{VaR} = 1.65 \sqrt{\sum_{m=1}^{3} \sum_{n=1}^{3} v_m \sigma_m v_n \sigma_n \text{corr}(m,n)}, \quad (16)
\]

where \(v_m\) and \(v_m\) are the cash flows distributed to three factor.

### 3. Bond Portfolio VaR Measure Model III: Principle Component Analysis and its VaR

The principle component analysis also is a linear structure model that tries to find the major linear factors to explain the volatility of research objective dependent variables. It is quite interesting to compare those linear models and find valuable consequences among those models such as the factor volatility explanation ability, the factor relationships, and the factor representations. Searching the maximum factor variance as transforming the variance covariance of dependent variables, we can find the characteristic vector and roots needed for finding the factor principle components. Supposed that the key rate variance covariance matrix is \(\Sigma\). The characteristic vector is \(c\) and the characteristic root is \(\lambda\). Then,

\[ \Sigma c = \lambda c, \quad (17) \]
Above equation expressed in terms of matrix operation, we get

$$\Sigma \begin{bmatrix}
  c_{11} & \ldots & c_{n1} \\
  \vdots & \ddots & \vdots \\
  c_{1n} & \ldots & c_{nn}
\end{bmatrix} = \begin{bmatrix}
  c_{11} & \ldots & c_{n1} \\
  \vdots & \ddots & \vdots \\
  c_{1n} & \ldots & c_{nn}
\end{bmatrix} \begin{bmatrix}
  \lambda_1 \\
  \vdots \\
  \lambda_n
\end{bmatrix}$$

(18)

$$\Sigma C = C \Lambda ,$$

(19)

where C is the characteristic matrix, and is orthogonal; thus

$$CC' = I ,$$

(20)

and so that

$$C' \Sigma C = C'C \Lambda = \Lambda .$$

(21)

Due to the principle component factor independence, we thus could find the 95% confidence level would be

$$\text{VaR} = 1.65 \sqrt{\sum_{i=1}^{n} v_i \sigma_i^f} ,$$

(22)

where vi is the ith principle component cash flow with respect to the one basis point change, $\sigma_i^f$ is the ith variance of the principle component, i.e. the characteristic matrix. The v_i can be found through the characteristic matrix transformation.

$$v_i = \sum_j c_{ij} * \text{MDur}_j * p_j ,$$

(23)

where c_{ij} is the jth element of the ith characteristic component vector matrix.

4. Bond Portfolio VaR Measure Model IV: Structure Equation Model

We adopt the confirmatory factor model (primarily SEM used). If there are n set of measures with $m_1, m_2, \ldots, m_n$ ($q = m_1 + m_2 + \ldots + m_n$) variables in each set respectively, then we can set
\[ X = \Lambda \eta + \varepsilon, \]  

(24)

where \( X \) is the different maturity yield data vector with order of \( q \) by one, \( \eta \) is the structure factor \( n \) by one vector, \( \Lambda \) is the structural coefficient matrix of \( X \) on \( \eta \) with order of \( q \) by \( n \), and \( \varepsilon \) is the error term with order of \( q \). Since we use current maturities 3m, 6m, 1y, 3y, 5y, 10y, 20y, and 30y as our key rate vertex, the paradigm of the model relationship can be displayed as the following figure showing. We should note that we need to do confirmatory factor analysis to find the appropriate factor connections with the observable variables.

As to the yield data retrieve, the US treasury bond yield curve data can be acquired from the web database of the US federal reserve bank. However, the yield data of different maturity Taiwan government bonds would be difficult to obtain since the Taiwan bond market is not active and public. We can find the bond data from Aremos databank or R.O.C. Over the Counter Security Exchange databank.

![Figure 1 Structural Equation Modeling Factor Analysis](image-url)
CHAPTER IV

YIELD CURVE MODEL APPLICATIONS OF US AND TAIWAN GOVERNMENT BONDS

US government bond market is the most efficient interest rate market in the world. Many financial institutes—banks, insurance, and investors—have made a large amount of interest rate transactions here to invest, hedge, speculate or arbitrage interest rate changes. US treasure bill, treasure note and treasure bond are the most traded securities with different maturities and by issuing them US government can finance its budget deficit or refinance its old debt.

On the other hand, since 1991 Taiwan government bond market has adopted bid-ask price auction, bond market in Taiwan will become more liquid and efficient. Its trading amount would increase more than before.

Historical different maturity interest rate data can be found from the website of the Federal Reserve Bank and as to Taiwan bond yield data, we can get the data from FSM databank of Aremos. Figure 2 and Figure 3 show respectively US and Taiwan bond yield timer series movement. Clearly the yield time series between them have some kind of correlations. In recent year 2002, the time series have the downward tendency when both of the government central banks wanted to stimulate their sloppy economy. While in late 1998, both of the countries experienced the growing economic, and Federal Reserve Bank suppressed the economy by uplifting its interest rate level in all interest rate term maturities. But in Taiwan we have started to suffer since late 1999, and the interest rates of all maturities were going down till recently. From the US interest rate time series, we also see the higher spread and volatility when the economic in the downward side—lower interest rate level— and the low volatility and spread while economic in the upward side—higher interest rate level.
Figure 2 US Bond Yield Time Series

Figure 3 Taiwan Bond Yield Time Series
1. Key Rate Duration Value at Risk

We first will use key rate duration method to measure the bond value at risk. Due to the much more different bond maturities, we will refer the key rate to RiskMetrics's key rate and use maturities- 3 month, 6 month, 1 year, 2 year, 3 year, 5 year, 7 year, 10 year, 20 year, and finally 30 year as our research key rates. Furthermore, JP Morgan RiskMetrics has provided its US bond market index. Thus, we will use this index as the base investment amount for the measure of bond value at risk.

The research data for US bond yield data collected form 8/8/1996 to 8/9/2002 are the yield to maturity data matched with our key rates i.e. the maturities as described above. On the other hand, Taiwan's long term bonds have been traded on and off. The data in Aremos is not enough and integrity. Thus, we must interpolate Taiwan's long term data for 5 year, 7 year, and 10 year maturity bond data. Unfortunately there are few data for the longer term such as the 15 year and above and thus we forgo the longer terms than 10 year. We primarily use spline interpolation method to form the time series of its bond data. In table1, we present the key rate variance covariance matrix along with the VaRs(%) of 95% confidence level for US and Taiwan bond yield data. It shows there are higher key rate variances from 2 to 10 year terms and higher correlations between relatively less term difference key rates in US bond data while in Taiwan we see that except for the 5 year maturity bond yield the, bond data exist some kind of higher correlation than US's bond data. The bond index i.e. the investment amount, doesn't need to be mapped into key rates since we already use its interest rate index structure like JP Morgan to analyze our key rate duration and value at risk. Therefore, with the key rate variance covariance and VaR(%) on hand and according to the bond index investment, we could find our bond portfolio value at risk. Table 2 is the value at risk figures of US and Taiwan bond yield based upon JP Morgan bond index. Obviously we see US bond yield
has bond VaR amount more than Taiwan bond because of the differences between the duration, the key rate volatility, and the correlation of the key rates. However, we should know that second order effect of the bond yield- convexity- would affect our risk measure as well.

With the 100 million on hand and bond index investment, we estimate around 0.031 and 0.018 million One-day VaR in US and Taiwan government bond market respectively. The diversification effect VaRs show the bond portfolio will have lower risk as the bond maturity difference becomes larger or as more of the different maturities involve in the bond investment. For instance, we see bond index position has VaR 0.031 less than the position 1, 2 and 3 while position 4 that is consisted of only two maturities involved: 3 month and 30 year but larger maturity difference has lower VaR of 0.013.

What if the bond cash flow can not match our key rate maturity? The cash flow mapping will be needed for variance equality or duration equality. We make an example of there kinds of bonds: 4% 1 year bond, 5% 3 year bond, and 6% 5 year bond and invest totally amount of 299.4 millions. According to the cash flow mapping (by principle of equality variances), we can get cash flow allocation as Table 3-1. The volatility weight is the base for the cash flow distribution between 3 year and 5 year key rate. Therefore 48.8% of 4 year cash flow will be allocated into 3 year key rate and then 51.2% into 5 year key rate. The VaR of 0.024 is the risk measure of the three bonds: 1, 3, and 5 year bond portfolio that also has less VaR figure than JP Morgan bond index Risk- VaR of 0.031. Since the bond cash flow key rates have large correlations, the bond portfolio consisted of fewer bonds might exhibit less risk as well. Table 3-2 is the cash position difference effect upon the bond portfolio VaR number. If the weight of cash investment puts more on the 5 year maturity
<table>
<thead>
<tr>
<th>Maturity</th>
<th>Spot Rate</th>
<th>VaR(%)</th>
<th>CM3m</th>
<th>CM6M</th>
<th>CM1Y</th>
<th>CM2Y</th>
<th>CM3Y</th>
<th>CM5Y</th>
<th>CM7Y</th>
<th>CM10Y</th>
<th>CM20Y</th>
<th>CM30Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM3m</td>
<td>4.45%</td>
<td>1.27%</td>
<td>1</td>
<td>0.980</td>
<td>0.978</td>
<td>0.925</td>
<td>0.882</td>
<td>0.881</td>
<td>0.785</td>
<td>0.623</td>
<td>0.494</td>
<td>0.467</td>
</tr>
<tr>
<td>CM6M</td>
<td>4.56%</td>
<td>1.23%</td>
<td>0.980</td>
<td>1</td>
<td>0.944</td>
<td>0.875</td>
<td>0.824</td>
<td>0.824</td>
<td>0.715</td>
<td>0.554</td>
<td>0.426</td>
<td>0.436</td>
</tr>
<tr>
<td>CM1Y</td>
<td>4.82%</td>
<td>1.27%</td>
<td>0.978</td>
<td>0.944</td>
<td>1</td>
<td>0.981</td>
<td>0.955</td>
<td>0.955</td>
<td>0.884</td>
<td>0.747</td>
<td>0.633</td>
<td>0.580</td>
</tr>
<tr>
<td>CM2Y</td>
<td>5.14%</td>
<td>1.15%</td>
<td>0.925</td>
<td>0.875</td>
<td>0.815</td>
<td>1</td>
<td>0.993</td>
<td>0.993</td>
<td>0.844</td>
<td>0.745</td>
<td>0.659</td>
<td></td>
</tr>
<tr>
<td>CM3Y</td>
<td>5.27%</td>
<td>1.02%</td>
<td>0.882</td>
<td>0.824</td>
<td>0.955</td>
<td>0.993</td>
<td>1</td>
<td>1.000</td>
<td>0.979</td>
<td>0.895</td>
<td>0.807</td>
<td>0.708</td>
</tr>
<tr>
<td>CM5Y</td>
<td>5.27%</td>
<td>1.02%</td>
<td>0.881</td>
<td>0.824</td>
<td>0.955</td>
<td>0.993</td>
<td>1.000</td>
<td>1</td>
<td>0.979</td>
<td>0.895</td>
<td>0.807</td>
<td>0.711</td>
</tr>
<tr>
<td>CM7Y</td>
<td>5.45%</td>
<td>0.83%</td>
<td>0.785</td>
<td>0.715</td>
<td>0.884</td>
<td>0.952</td>
<td>0.979</td>
<td>1</td>
<td>0.963</td>
<td>0.894</td>
<td>0.791</td>
<td></td>
</tr>
<tr>
<td>CM10Y</td>
<td>5.65%</td>
<td>0.65%</td>
<td>0.623</td>
<td>0.554</td>
<td>0.747</td>
<td>0.844</td>
<td>0.895</td>
<td>0.895</td>
<td>1</td>
<td>0.972</td>
<td>0.880</td>
<td></td>
</tr>
<tr>
<td>CM20Y</td>
<td>6.10%</td>
<td>0.51%</td>
<td>0.494</td>
<td>0.426</td>
<td>0.633</td>
<td>0.745</td>
<td>0.807</td>
<td>0.807</td>
<td>0.894</td>
<td>1</td>
<td>0.906</td>
<td></td>
</tr>
<tr>
<td>CM30Y</td>
<td>5.90%</td>
<td>0.55%</td>
<td>0.467</td>
<td>0.436</td>
<td>0.580</td>
<td>0.659</td>
<td>0.708</td>
<td>0.711</td>
<td>0.791</td>
<td>0.880</td>
<td>1</td>
<td>0.906</td>
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</table>

II. TAIWAN BOND YIELD

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Spot Rate</th>
<th>VaR(%)</th>
<th>CM3m</th>
<th>CM6M</th>
<th>CM1Y</th>
<th>CM2Y</th>
<th>CM3Y</th>
<th>CM5Y</th>
<th>CM7Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM3m</td>
<td>4.89%</td>
<td>1.15%</td>
<td>1</td>
<td>0.985338</td>
<td>0.969043</td>
<td>0.970953</td>
<td>0.975485</td>
<td>-0.08923</td>
<td>0.80115</td>
</tr>
<tr>
<td>CM6M</td>
<td>5.15%</td>
<td>1.26%</td>
<td>0.985338</td>
<td>1</td>
<td>0.994516</td>
<td>0.995299</td>
<td>0.996221</td>
<td>-0.01791</td>
<td>0.859358</td>
</tr>
<tr>
<td>CM1Y</td>
<td>5.51%</td>
<td>1.33%</td>
<td>0.999043</td>
<td>0.994516</td>
<td>1</td>
<td>0.999701</td>
<td>0.999315</td>
<td>0.026325</td>
<td>0.887505</td>
</tr>
<tr>
<td>CM2Y</td>
<td>5.57%</td>
<td>1.34%</td>
<td>0.970953</td>
<td>0.995299</td>
<td>0.999701</td>
<td>1</td>
<td>0.999663</td>
<td>0.017146</td>
<td>0.886145</td>
</tr>
<tr>
<td>CM3Y</td>
<td>5.59%</td>
<td>1.34%</td>
<td>0.975485</td>
<td>0.996221</td>
<td>0.999315</td>
<td>0.999663</td>
<td>1</td>
<td>0.005279</td>
<td>0.882482</td>
</tr>
<tr>
<td>CM5Y</td>
<td>7.75%</td>
<td>1.85%</td>
<td>-0.08923</td>
<td>-0.01791</td>
<td>0.026325</td>
<td>0.017146</td>
<td>0.005279</td>
<td>1</td>
<td>0.093104</td>
</tr>
<tr>
<td>CM7Y</td>
<td>6.66%</td>
<td>1.51%</td>
<td>0.80115</td>
<td>0.859358</td>
<td>0.887505</td>
<td>0.886145</td>
<td>0.882482</td>
<td>0.093104</td>
<td>1</td>
</tr>
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</table>
### TABLE 2 JP MORGAN BOND INDEX INVESTMENT VALUE AT RISK

**(DAILY BASE, 95% CONFIDENCE LEVEL)**

#### (1) US GOVERNMENT BOND

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Spot Rate</th>
<th>VaR(%)</th>
<th>债券指數部位(M$m)</th>
<th>指數的部位</th>
<th>部位 1</th>
<th>部位 2</th>
<th>部位 3</th>
<th>部位 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM3m</td>
<td>4.45%</td>
<td>0.0831%</td>
<td>1.73</td>
<td>0.00433</td>
<td>0.87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM6m</td>
<td>4.56%</td>
<td>0.0243%</td>
<td>3.19</td>
<td>0.01597</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM1Y</td>
<td>4.82%</td>
<td>0.0838%</td>
<td>17.91</td>
<td>0.17907</td>
<td>0.66</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM2Y</td>
<td>5.14%</td>
<td>0.1007%</td>
<td>31.85</td>
<td>0.63699</td>
<td>0.588</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM3Y</td>
<td>5.27%</td>
<td>0.1028%</td>
<td>19.75</td>
<td>0.59261</td>
<td>0.47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM5Y</td>
<td>5.27%</td>
<td>0.1078%</td>
<td>9.77</td>
<td>0.48871</td>
<td>0.53</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM7Y</td>
<td>5.45%</td>
<td>0.1036%</td>
<td>5.79</td>
<td>0.40495</td>
<td>0.412</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM10Y</td>
<td>5.65%</td>
<td>0.1021%</td>
<td>4.28</td>
<td>0.42842</td>
<td>0.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM20Y</td>
<td>6.10%</td>
<td>0.0858%</td>
<td>4.04</td>
<td>0.80811</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>CM30Y</td>
<td>5.90%</td>
<td>0.0328%</td>
<td>1.68</td>
<td>0.50410</td>
<td>0.13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Duration= | 4.06 | 4.06 | 4.06 | 4.06 | 4.06 |
| Undiversified VaR= | 0.0945 | 0.1055 | 0.1019 | 0.0900 | 0.0767 |
| VaR= | 0.0313 | 0.0375 | 0.0366 | 0.0358 | 0.0130 |
| Diversification Effect= | 0.0632 | 0.0680 | 0.0653 | 0.0543 | 0.0637 |
| Diversification Effect (%)= | 66.86% | 64.49% | 64.11% | 60.25% | 83.09% |

#### (2) TAIWAN GOVERNMENT BOND

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Spot Rate</th>
<th>VaR(%)</th>
<th>Key Rate Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bond Index</td>
<td>Pos1</td>
<td>Pos2</td>
</tr>
<tr>
<td>CM3m</td>
<td>4.45%</td>
<td>0.0831%</td>
<td>1.92</td>
</tr>
<tr>
<td>CM6m</td>
<td>4.56%</td>
<td>0.0243%</td>
<td>3.55</td>
</tr>
<tr>
<td>CM1Y</td>
<td>4.82%</td>
<td>0.0838%</td>
<td>19.90</td>
</tr>
<tr>
<td>CM2Y</td>
<td>5.14%</td>
<td>0.1007%</td>
<td>35.39</td>
</tr>
<tr>
<td>CM3Y</td>
<td>5.27%</td>
<td>0.1028%</td>
<td>21.95</td>
</tr>
<tr>
<td>CM5Y</td>
<td>5.27%</td>
<td>0.1078%</td>
<td>10.86</td>
</tr>
<tr>
<td>CM7Y</td>
<td>5.45%</td>
<td>0.1036%</td>
<td>6.43</td>
</tr>
</tbody>
</table>

| Duration= | 2.5809 | 1.5300 | 1.5300 | 1.8650 | 1.1145 |
| Undiversified VaR= | 0.0957 | 0.0928 | 0.0567 | 0.0915 | 0.0857 |
| VaR= | 0.0180 | 0.0122 | 0.0116 | 0.0159 | 0.0090 |
| Diversification Effect= | 0.0777 | 0.0806 | 0.0451 | 0.0756 | 0.0767 |
| Diversification Effect (%)= | 81.15% | 86.86% | 79.54% | 82.65% | 89.50% |
TABLE 3-1 BOND PORTFOLIO CASH FLOW DISTRIBUTION ON THE KEY RATE BY VARIANCE EQUALITY AND ITS VALUE AT RISK

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Spot Rate</th>
<th>VaR(%)</th>
<th>CF 1yr</th>
<th>CF 3yr</th>
<th>CF 5yr</th>
<th>CashVaR</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM3m</td>
<td>4.45%</td>
<td>0.083%</td>
<td>1yr</td>
<td>3yr</td>
<td>5yr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM6m</td>
<td>4.56%</td>
<td>0.024%</td>
<td>103</td>
<td>6</td>
<td>114.000</td>
<td>108.75</td>
<td>108.75</td>
</tr>
<tr>
<td>CM1Y</td>
<td>4.82%</td>
<td>0.084%</td>
<td>5</td>
<td>6</td>
<td>110.000</td>
<td>108.75</td>
<td>0.09115</td>
</tr>
<tr>
<td>CM2Y</td>
<td>5.14%</td>
<td>0.101%</td>
<td>5</td>
<td>6</td>
<td>111.000</td>
<td>9.98</td>
<td>9.98</td>
</tr>
<tr>
<td>CM3Y</td>
<td>5.27%</td>
<td>0.103%</td>
<td>105</td>
<td>6</td>
<td>111.000</td>
<td>95.67</td>
<td>98.1</td>
</tr>
<tr>
<td>CM4Y</td>
<td>5.27%</td>
<td>0.105%</td>
<td>106</td>
<td>6</td>
<td>106.000</td>
<td>84.91</td>
<td>0.08793</td>
</tr>
<tr>
<td>CM5Y</td>
<td>5.27%</td>
<td>0.108%</td>
<td>106</td>
<td>6</td>
<td>106.000</td>
<td>82.44</td>
<td></td>
</tr>
<tr>
<td>CM7Y</td>
<td>5.45%</td>
<td>0.104%</td>
<td>108</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM10Y</td>
<td>5.65%</td>
<td>0.102%</td>
<td>106</td>
<td>106</td>
<td>106.000</td>
<td>82.44</td>
<td>0.08793</td>
</tr>
<tr>
<td>CM20Y</td>
<td>6.10%</td>
<td>0.086%</td>
<td>106</td>
<td>106</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM30Y</td>
<td>5.90%</td>
<td>0.033%</td>
<td>106</td>
<td>106</td>
<td></td>
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<td></td>
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<tr>
<td>Total VaR &amp; Weight</td>
<td>0.48814</td>
<td>0.29004</td>
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<tr>
<td>VaR</td>
<td>0.07248</td>
<td>0.07289</td>
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<tr>
<td>%VaR</td>
<td>0.0242%</td>
<td>0.0243%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Diversified VaR</td>
<td>0.751</td>
<td>0.750</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1y CF</td>
<td>108.480</td>
<td>107.480</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2y CF</td>
<td>9.924</td>
<td>9.924</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3y CF</td>
<td>97.283</td>
<td>97.283</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4y CF</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5y CF</td>
<td>83.707</td>
<td>84.707</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tot CF</td>
<td>299.394</td>
<td>299.394</td>
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TABLE 3-2 KEY RATE CASH FLOW DIFFERENCE EFFECT UPON VALUE AT RISK

<table>
<thead>
<tr>
<th>CF Diff B/t 1 yr and 5yr</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
<th>7.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR</td>
<td>0.07248</td>
<td>0.07289</td>
<td>0.07329</td>
<td>0.07370</td>
<td>0.07411</td>
<td>0.07451</td>
<td>0.07492</td>
</tr>
<tr>
<td>%VaR</td>
<td>0.0242%</td>
<td>0.0243%</td>
<td>0.0245%</td>
<td>0.0246%</td>
<td>0.0248%</td>
<td>0.0249%</td>
<td>0.0250%</td>
</tr>
<tr>
<td>Diversified VaR</td>
<td>0.751</td>
<td>0.750</td>
<td>0.749</td>
<td>0.748</td>
<td>0.746</td>
<td>0.745</td>
<td>0.744</td>
</tr>
<tr>
<td>1y CF</td>
<td>108.480</td>
<td>107.480</td>
<td>105.480</td>
<td>102.480</td>
<td>98.480</td>
<td>93.480</td>
<td>87.480</td>
</tr>
<tr>
<td>3y CF</td>
<td>97.283</td>
<td>97.283</td>
<td>97.283</td>
<td>97.283</td>
<td>97.283</td>
<td>97.283</td>
<td>97.283</td>
</tr>
<tr>
<td>4y CF</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5y CF</td>
<td>83.707</td>
<td>84.707</td>
<td>86.707</td>
<td>89.707</td>
<td>93.707</td>
<td>98.707</td>
<td>104.707</td>
</tr>
<tr>
<td>Tot CF</td>
<td>299.394</td>
<td>299.394</td>
<td>299.394</td>
<td>299.394</td>
<td>299.394</td>
<td>299.394</td>
<td>299.394</td>
</tr>
</tbody>
</table>

bond, we will get more VaR figure. The reason is coming from the 5 year higher volatility and its correlation with 3 year maturity.

2. Level, Slope, and Curvature Duration Value at Risk

We first review the yield curve term structure change. We will find that in early 1998 US interest rate structure existed higher level term structure, and relatively the lower structure slope and curvature. Contrary to early 1998, we will find that in early 1999 US interest rate structure existed
lower level tendency and higher curvature. There is some sort of evidence showing that from early 1998 to early 1999 US seemed to experience the slightly recession and since then its economic growth has rebounded gradually. In late 1999 we can see the high level, high slope, and high curvature interest rate term structure. The situation was resulted from the high inflation and overheated money in the stock market. To restrain its overheated economic, US Federal Reserve Bank therefore uplifted its discount interest rate and hoped to cease the economic overheated phenomena. To be more comprehensive, the level of the interest rate term structure would reflect the inflation index, the slope of the interest rate term structure would reflect the economic business cycle, and the curvature of the interest rate term structure would reflect economic volatility. Figure 4 presents the yield curve change between early 1998 and middle of 2002. It shows level, slope, and curvature movements as the time passes. Clearly in US term structure movement the slope and level have the negative relationship, i.e. when in high slope, the economic will become sloppy and when in high level the economic will get stronger. We also use the three factor yield curve model to approximate the yield curve term structure of 08/09/2002. The optimal vertex (hump) for the US interest rate term structure is around 2.03 years and we get the fine term structure forecast as Figure 4 shows.

While in Taiwan, Figure 5 shows the slightly negative relationship between level and slope. In recent year the longer term interest rates in Taiwan have came down more than US's long term interest rates. Investor such as banks, insurers and other financial institutes speculate the stagnant economic will be coming for the future years as people in Taiwan prefer to invest overseas for their cheaper labor force or product material. The optimal vertex (hump) for Taiwan interest rate term structure is around 0.88 year and we get the fine term structure forecast as well in Figure 5. The shorter term of vertex in Taiwan is partly resulted from its shorter interest rate term structure and
Figure 4 US Bond Yield Term Structure Movement

Figure 5 Taiwan Bond Yield Term Structure Movement
partly resulted from its slope and curvature term structure components.

In order to measure the bond value at risk by the three factor yield curve model, we must estimate the three factors' parameters. Table 4 provides the level, slope, and curvature correlations, and variance covariance information calculated using the whole period yield curve data. Recalling from figure 4, we can see negative correlation between the level and the slope while there is positive correlation between the slope and the curvature. This is correspondent to our correlation figures shown in table 4 for US bond yield data. On the other hand, in Taiwan there are negative relationship between the slope and the curvature factors and positive relationship between the level and the curvature as well. The relationship of the level and the curvature is disputable from the bond data of the US and Taiwan that show the opposite direction of the level and the curvature relationship as presented in Table 4. This might be explained as the economic growing, people will prefer the longer term borrowing and short term investment since people predict the lower short rate and higher long rate will go down and as the economic recessing, people will do oppositely in US bond market. While in Taiwan, people will predict the going-up long term rate as the economic growing and react differently as US people do.

As to the bond value at risk measure, like the key rate duration, we must find the duration contributed to specifically the level, the slope, and the curvature. Since the level is fixed at one, we can clearly recognize it has the duration equal to our cash flow. The slope and the curvature formed by our three factor model would be necessarily to be estimated and optimized. With the least square forecasting error method, we can obtain the table 5 that shows both of the US and Taiwan estimated three factor model parameters. The tau is our optimal term structure vertex (hump), the level remains one, and the slope and the curvature are estimated by tau that is 2.03 for US and 0.88 for Taiwan.
With the three factor parameters and its variance covariance on hand, we then can estimate the VaR figure of bond yield change for both of US and Taiwan. Table 6 shows the three factor model measure VaR. US bond data again has higher VaR figure than Taiwan's. We also provide the VaR sensitivity measures of the three factors by computing its VaRdelta and VaRbeta that measure the VaR change with respect to the three factor changes. As the table shows, the curvature factor has the most sensitive VaR for both US and Taiwan bond yield. The slope and the level have little effect upon VaR figure sensitivity for US bond yield while having great effect upon VaR figure sensitivity for Taiwan bond yield. In US while in higher level or lower slope, there is higher VaR sensitivity. In Taiwan however there is opposite VaR sensitivity to US bond yield happened.

**TABLE 4 LEVEL SLOPE CURVATURE CORRELATIONS**

(1) US Bond Yield

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Level</th>
<th>Slope</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>1</td>
<td>-0.23986</td>
<td>0.25642</td>
</tr>
<tr>
<td>Slope</td>
<td>-0.23986</td>
<td>1</td>
<td>0.25681</td>
</tr>
<tr>
<td>Curvature</td>
<td>0.25642</td>
<td>0.25681</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance Covariance</th>
<th>Level</th>
<th>Slope</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>1.98E-05</td>
<td>-1.2E-05</td>
<td>0.000177</td>
</tr>
<tr>
<td>Slope</td>
<td>-1.2E-05</td>
<td>0.000177</td>
<td>1</td>
</tr>
<tr>
<td>Curvature</td>
<td>1.9E-05</td>
<td>5.79E-05</td>
<td>0.000359</td>
</tr>
</tbody>
</table>

(2) Taiwan Bond Yield

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Level</th>
<th>Slope</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>1</td>
<td>-0.81586</td>
<td>-0.8159</td>
</tr>
<tr>
<td>Slope</td>
<td>-0.81586</td>
<td>1</td>
<td>0.982740795</td>
</tr>
<tr>
<td>Curvature</td>
<td>-0.8159</td>
<td>0.982740795</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance Covariance</th>
<th>Level</th>
<th>Slope</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>8.6E-08</td>
<td>-8.3E-08</td>
<td>-3.5E-07</td>
</tr>
<tr>
<td>Slope</td>
<td>-8.3E-08</td>
<td>1.44E-07</td>
<td>3.4E-07</td>
</tr>
<tr>
<td>Curvature</td>
<td>-3.5E-07</td>
<td>3.4E-07</td>
<td>1.73052E-06</td>
</tr>
</tbody>
</table>
### Table 5 Level, Slope, Curvature Parameter Estimates

**1) US Bond Yield**

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Level</th>
<th>Slope</th>
<th>Curve</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 CM3m</td>
<td>1</td>
<td>0.94</td>
<td>0.057</td>
<td>1.73</td>
</tr>
<tr>
<td>0.5 CM6M</td>
<td>1</td>
<td>0.89</td>
<td>0.105</td>
<td>3.19</td>
</tr>
<tr>
<td>1 CM1Y</td>
<td>1</td>
<td>0.79</td>
<td>0.178</td>
<td>17.91</td>
</tr>
<tr>
<td>2 CM2Y</td>
<td>1</td>
<td>0.64</td>
<td>0.263</td>
<td>31.85</td>
</tr>
<tr>
<td>3 CM3Y</td>
<td>1</td>
<td>0.52</td>
<td>0.294</td>
<td>19.75</td>
</tr>
<tr>
<td>5 CM5Y</td>
<td>1</td>
<td>0.37</td>
<td>0.286</td>
<td>9.77</td>
</tr>
<tr>
<td>7 CM7Y</td>
<td>1</td>
<td>0.28</td>
<td>0.249</td>
<td>5.79</td>
</tr>
<tr>
<td>10 CM10Y</td>
<td>1</td>
<td>0.20</td>
<td>0.194</td>
<td>4.28</td>
</tr>
<tr>
<td>20 CM20Y</td>
<td>1</td>
<td>0.10</td>
<td>0.102</td>
<td>4.04</td>
</tr>
<tr>
<td>30 CM30Y</td>
<td>1</td>
<td>0.07</td>
<td>0.068</td>
<td>1.68</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**2) Taiwan Bond Yield**

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Level</th>
<th>Slope</th>
<th>Curve</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 CM3m</td>
<td>1</td>
<td>0.87</td>
<td>0.118</td>
<td>1.73</td>
</tr>
<tr>
<td>0.5 CM6M</td>
<td>1</td>
<td>0.76</td>
<td>0.197</td>
<td>3.19</td>
</tr>
<tr>
<td>1 CM1Y</td>
<td>1</td>
<td>0.60</td>
<td>0.277</td>
<td>17.91</td>
</tr>
<tr>
<td>2 CM2Y</td>
<td>1</td>
<td>0.39</td>
<td>0.291</td>
<td>31.85</td>
</tr>
<tr>
<td>3 CM3Y</td>
<td>1</td>
<td>0.28</td>
<td>0.250</td>
<td>19.75</td>
</tr>
<tr>
<td>5 CM5Y</td>
<td>1</td>
<td>0.17</td>
<td>0.171</td>
<td>9.77</td>
</tr>
<tr>
<td>7 CM7Y</td>
<td>1</td>
<td>0.12</td>
<td>0.125</td>
<td>5.79</td>
</tr>
<tr>
<td>10 CM10Y</td>
<td>1</td>
<td>0.09</td>
<td>0.087</td>
<td>4.28</td>
</tr>
<tr>
<td>20 CM20Y</td>
<td>1</td>
<td>0.04</td>
<td>0.044</td>
<td>4.04</td>
</tr>
<tr>
<td>30 CM30Y</td>
<td>1</td>
<td>0.03</td>
<td>0.029</td>
<td>1.68</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 6 LEVEL, SLOPE, CURVATURE DURATION VALUE AT RISK

<table>
<thead>
<tr>
<th></th>
<th>dL,dS,dC Covariance</th>
<th>LSC Cash Flow</th>
<th>VaR Delta</th>
<th>VaR Component</th>
<th>VaR Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>US Bond Yield</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dLevel</td>
<td>7.16929E-08 -6.59651E-08 -4.95957E-08</td>
<td>100</td>
<td>0.00007</td>
<td>0.0072</td>
<td>8.28%</td>
</tr>
<tr>
<td>dSlope</td>
<td>-6.59651E-08 1.76389E-07 -3.23266E-07</td>
<td>55.83317641</td>
<td>-0.00013</td>
<td>-0.0075</td>
<td>-8.57%</td>
</tr>
<tr>
<td>dCurvature</td>
<td>-4.95957E-08 -3.23266E-07 6.12824E-06</td>
<td>23.39896841</td>
<td>0.00375</td>
<td>0.0877</td>
<td>100.29%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0874</td>
<td>100.00%</td>
</tr>
<tr>
<td><strong>Taiwan Bond Yield</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dLevel</td>
<td>8.59625E-08 -8.25566E-08 -3.46302E-07</td>
<td>100</td>
<td>-0.00028</td>
<td>-0.0277</td>
<td>-68.42%</td>
</tr>
<tr>
<td>dSlope</td>
<td>-8.25566E-08 1.43756E-07 3.40203E-07</td>
<td>55.83317641</td>
<td>0.00052</td>
<td>0.0290</td>
<td>71.74%</td>
</tr>
<tr>
<td>dCurvature</td>
<td>-3.46302E-07 3.40203E-07 1.73052E-06</td>
<td>23.39896841</td>
<td>0.00167</td>
<td>0.0391</td>
<td>96.67%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0405</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

**VaR = 0.0874**

<table>
<thead>
<tr>
<th></th>
<th>dL,dS,dC Covariance</th>
<th>LSC Cash Flow</th>
<th>VaR Delta</th>
<th>VaR Component</th>
<th>VaR Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Taiwan Bond Yield</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dLevel</td>
<td>8.59625E-08 -8.25566E-08 -3.46302E-07</td>
<td>100</td>
<td>-0.00028</td>
<td>-0.0277</td>
<td>-68.42%</td>
</tr>
<tr>
<td>dSlope</td>
<td>-8.25566E-08 1.43756E-07 3.40203E-07</td>
<td>55.83317641</td>
<td>0.00052</td>
<td>0.0290</td>
<td>71.74%</td>
</tr>
<tr>
<td>dCurvature</td>
<td>-3.46302E-07 3.40203E-07 1.73052E-06</td>
<td>23.39896841</td>
<td>0.00167</td>
<td>0.0391</td>
<td>96.67%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0405</td>
<td>100.00%</td>
</tr>
<tr>
<td><strong>VaR = 0.0405</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 3. Principal component duration value at risk

Principal component analysis (PCA) uses the characteristic value of key rate variance-covariance and correlation and then transforms the structure of variables into primary several components while keeping the original total variance or maximizing the transformation total variance. After variable transformation by PCA, we should know the primary components will be independent of each other and normally there will be two or three primary components accounting for the variable variance. Compared to key rate and three factor yield curve model, the PCA will have the advantage of the factor independence that is helpful for real investment analysis since we don't need to care about the consequential factor relationship risk. However, the primary
components are not easy to be described specifically. Therefore, PCA still has been considered as the theory purpose tool. On the other hand, unlike the three factor model that finds the optimal tau by minimizing the forecast error, PCA will find their eigen value and vector by maximizing the score variance. Thus, PCA wouldn't be used for the time series prediction but used for the system analysis that uses the whole information for the parameter estimates such as the eigen value and vector computation.

In table 7, we present the eigen value and eigen vector of the variance and covariance of the key rate and the PCA duration. Further using the PCA duration alone without its variance and covariance structure, we then can measure the PCA bond yield VaR figure for both US and Taiwan bond yield since PCA durations are independent to each other.

Table 8 uses the PCA duration comprised of the maturity, key rate duration that is consisted of cash flow and maturity, and eigen volatility to measure the bond yield VaR for both of US and Taiwan. Three of the components construct 94% of the term structure variance for US bond yield while constructing 99.7% of the term structure variance for Taiwan bond yield. Surely, we should know that we have more key rate maturities in US bond data and less term maturities in Taiwan bond data in our research. The higher US bond yield VaR figure will mean its interest rate system risk greater than Taiwan's but it doesn't relate to the short term conditional VaR of US or Taiwan's bond yield. The larger PCA system VaR figure will disclose Taiwan's longer time level investment risk as well, i.e. if our analysis is based on the one month VaR, the system PCA VaR might present the higher VaR figure than US's. So we propose if the interest rate trend is much more distanced in Taiwan, the VaR figure measured by longer time level e.g. one month or one year, would show higher bond yield change risk in Taiwan than in US.
### Table 7: The Eigenvalue and Eigenvector of Variance Covariance Interest Rate Term Structure

#### (1) US Bond Yield Eigenvalue and Eigen Vector

<table>
<thead>
<tr>
<th>Eigen Value $\lambda$</th>
<th>Eigen Vol</th>
<th>Eigen Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2572E-06</td>
<td>0.150%</td>
<td>-0.84630 0.45940 0.01500 0.04440 -0.21820 0.01970 0.01390 -0.00847 -0.01690</td>
</tr>
<tr>
<td>2.5013E-07</td>
<td>0.050%</td>
<td>-0.03260 -0.00649 -0.22080 -0.41530 -0.04740 -0.07370 -0.87740 0.00186 0.01070</td>
</tr>
<tr>
<td>1.0813E-07</td>
<td>0.033%</td>
<td>0.29710 -0.29430 -0.24200 -0.26850 -0.00294 0.79120 -0.21000 0.05880 0.13920 0.03830</td>
</tr>
<tr>
<td>6.0233E-08</td>
<td>0.025%</td>
<td>0.39310 -0.08600 -0.29910 0.11550 -0.09270 0.04090 0.75060 -0.04400 -0.39950 -0.00770</td>
</tr>
<tr>
<td>4.3087E-08</td>
<td>0.021%</td>
<td>0.39920 -0.01160 -0.18890 0.53910 -0.24560 -0.16800 -0.25860 0.02240 0.23330 0.55120</td>
</tr>
<tr>
<td>4.0555E-08</td>
<td>0.020%</td>
<td>0.42430 0.00965 -0.30070 -0.10020 0.07360 -0.34930 -0.05480 0.01140 0.46630 -0.60940</td>
</tr>
<tr>
<td>1.5514E-08</td>
<td>0.012%</td>
<td>0.40840 0.13120 0.04320 -0.08330 0.14250 -0.12760 -0.51760 -0.00220 -0.69890 -0.11900</td>
</tr>
<tr>
<td>1.4059E-08</td>
<td>0.012%</td>
<td>0.38220 0.27280 0.32020 -0.52810 0.29810 -0.12670 0.20130 -0.02290 0.21490 0.45390</td>
</tr>
<tr>
<td>1.3099E-08</td>
<td>0.011%</td>
<td>0.28780 0.31030 0.63920 0.34840 -0.22600 0.34320 0.08980 -0.02530 0.10380 -0.32050</td>
</tr>
<tr>
<td>5.1686E-09</td>
<td>0.007%</td>
<td>0.01000 0.02370 0.04290 -0.39720 -0.76880 -0.14840 -0.01560 0.47190 -0.06060 0.00769</td>
</tr>
</tbody>
</table>

#### Sum of $\lambda a'1a1=1$

| 0.99997 1.00003 1.00006 1.00008 0.99999 0.99999 1.00001 1.00008 |

#### (2) Taiwan Bond Yield Eigenvalue and Eigen Vector

<table>
<thead>
<tr>
<th>Eigen Value $\lambda$</th>
<th>Eigen Vol</th>
<th>Eigen Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E-03</td>
<td>3.165%</td>
<td>-0.35050 -0.07230 -0.33920 0.81320 0.28180 -0.06780 0.10750</td>
</tr>
<tr>
<td>3.46E-04</td>
<td>1.861%</td>
<td>-0.39330 -0.02900 -0.22210 0.05750 -0.88340 0.03100 -0.10280</td>
</tr>
<tr>
<td>4.47E-05</td>
<td>0.669%</td>
<td>-0.41900 0.00229 -0.13150 -0.36360 0.16350 -0.80430 0.03540</td>
</tr>
<tr>
<td>3.02E-06</td>
<td>0.174%</td>
<td>-0.42140 -0.00437 -0.13590 -0.33850 0.13630 0.45250 0.68270</td>
</tr>
<tr>
<td>4.11E-07</td>
<td>0.064%</td>
<td>-0.42200 -0.01310 -0.15020 -0.22640 0.30770 0.37780 -0.71450</td>
</tr>
<tr>
<td>5.10E-08</td>
<td>0.023%</td>
<td>-0.01240 0.99440 -0.09570 0.04380 0.00016 0.00506 -0.00200</td>
</tr>
<tr>
<td>1.78E-08</td>
<td>0.013%</td>
<td>-0.43720 0.07070 0.87640 0.18820 -0.01640 -0.00364 0.00405</td>
</tr>
</tbody>
</table>

#### Sum of $\lambda a'1a1=1$

| 0.8089 0.9951 0.2319 0.9646 0.9998 1.0000 1.0000 |

#### Sum of $\lambda a'1a2=0$

| 0.00004 0.00003 0.00001 0.00000 0.00000 0.00004 -0.00005 -0.00002 |
### TABLE 8 PCA DURATION AND VARS

#### (1) US Bond Yield

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Spot Rate</th>
<th>VaR(%)</th>
<th>Bond Index</th>
<th>Key Rate Duration</th>
<th>EigenValue = \lambda</th>
<th>PC Vol % Explained</th>
<th>% Vol Explained</th>
<th>Cummulated K*EigVol PC Duration</th>
<th>PC Duration^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM3m</td>
<td>4.451%</td>
<td>0.0831%</td>
<td>1.73</td>
<td>0.004</td>
<td>2.26E-06</td>
<td>0.1502%</td>
<td>80.411%</td>
<td>80.41%</td>
<td>6.50E-06</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.93E-06</td>
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<td>0.0208%</td>
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VaR= 4.30E-02

#### (2) Taiwan Bond Yield

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<th>VaR(%)</th>
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<th>Key Rate Duration</th>
<th>EigenValue = \lambda</th>
<th>PC Vol % Explained</th>
<th>% Vol Explained</th>
<th>Cummulated K*EigVol PC Duration</th>
<th>PC Duration^2</th>
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<td>0.02%</td>
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Total= 100.00 2.58 0.0013963 VaR= 0.03136
4. Structural Equation Yield Curve Model

Principal components analysis is to explore the data reducing process that is to find the primary component variances to maximize the total variances after variable transformation. Unlike PCA exploring the primary components with entire factor loading estimates, structural equation modeling (SEM) tries to control over the specifications of the constructs with partial factor loadings estimates that are the measurement model of the SEM, i.e. the construct (factor) measurement. Thus in our analysis of the SEM of the interest rate term structure, we will subjectively try to find the controlled constructs that can be measured by our indicators- the structural key rates with different maturities. We will use LISREL as our SEM program solution. Several steps will be involved in our interest rate term structure SEM mentioned as follows:

Step One: Find Construct Measurement

Considering our 10 key rates for US bond data ranging from 3 months to 30 years, and from the LISREL: the Modification Indices, we suggest splitting the term structure into short term, median term, and long term constructs as the table 9-(a) shows for our SEM confirmatory factor analysis. As to Taiwan bond yield, after the factor add and delete modification and chi-square and root mean square residual index examination, we will use three constructs consisted of the key rate ranging from 3 months to 7 years- construct one has 3 month, and 6 month maturities, construct two has 1 year and 2 year maturities, and finally construct three has 3 year, 5 year, and 7 year maturities as shown in table 9-(b).

Step Two: Data Input

The SEM will try to search the appropriate relationship (loadings) of the factors and indicators. Hence the data input for the SEM model estimation is the variance covariance decomposition of the covariance matrix or the correlation matrix. Whereas the correlation has been
used as usual for its measure unit, we should use the covariance instead as the input data for the variance research need of our value at risk analysis.

Step Three: Optimal Model Estimation

Like the multivariate data analysis such as the linear multivariate regression, multivariate logit or probit regression, and even the exploratory factor analysis and principal component analysis, the parameters of the SEM will become optimized by ordinary least square (OLS) or maximum likelihood estimator (MLE). Research has shown that MLE is superior to OLS measure for its general purpose estimate such as the consistency, asymptotic normality, asymptotic efficiency, and invariance. Herein we will use MLE as our optimization tool for SEM parameter (factor loading) estimation. Figure 6 and Figure 7 present the SEM estimation consequences for US and Taiwan bond yield respectively.

<table>
<thead>
<tr>
<th>TABLE 9 CONFIRMATORY FACTOR ANALYSIS SEM MEASURE MODEL NOTATIONS</th>
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(a) US Constructs

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<th>Exogenous Indicator (Key Rates)</th>
<th>Exogenous Constructs</th>
<th>Error</th>
</tr>
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<tbody>
<tr>
<td>X1(CF3m)</td>
<td>$\lambda_{11} \xi_{1}$</td>
<td>$\delta_{1}$</td>
</tr>
<tr>
<td>X2(CF6m)</td>
<td>$\lambda_{21} \xi_{1}$</td>
<td>$\delta_{2}$</td>
</tr>
<tr>
<td>X3(CF1y)</td>
<td>$\lambda_{31} \xi_{1}$</td>
<td>$\delta_{3}$</td>
</tr>
<tr>
<td>X4(CF2y)</td>
<td>$\lambda_{12} \xi_{2}$</td>
<td>$\delta_{4}$</td>
</tr>
<tr>
<td>X5(CF3y)</td>
<td>$\lambda_{22} \xi_{2}$</td>
<td>$\delta_{5}$</td>
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<tr>
<td>X6(CF5y)</td>
<td>$\lambda_{32} \xi_{2}$</td>
<td>$\delta_{6}$</td>
</tr>
<tr>
<td>X7(CF7y)</td>
<td>$\lambda_{13} \xi_{3}$</td>
<td>$\delta_{7}$</td>
</tr>
<tr>
<td>X8(CF10y)</td>
<td>$\lambda_{23} \xi_{3}$</td>
<td>$\delta_{8}$</td>
</tr>
<tr>
<td>X9(CF20y)</td>
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<td>$\delta_{9}$</td>
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<tr>
<td>X10(CF30y)</td>
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(b) Taiwan Constructs

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<th>Exogenous Indicator (Key Rates)</th>
<th>Exogenous Constructs</th>
<th>Error</th>
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<td>X1(CF3m)</td>
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<td>X4(CF2y)</td>
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<td>X9(CF20y)</td>
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<td>$\delta_{9}$</td>
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Chi-Square=6756.40, df=32, P-value=0.00000, RMSEA=0.447

Figure 6 Interest Rate Term Structure Structural Equation Modeling of US Bond Yield

Chi-Square=1430.58, df=11, P-value=0.00000, RMSEA=0.350

Figure 7 Interest Rate Term Structure Structural Equation Modeling of Taiwan Bond Yield
<table>
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<tr>
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<th>(a) US Bond Yield t Statistics</th>
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<th>(b) Taiwan Bond Yield t Statistics</th>
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<td></td>
<td>42.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>21.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM30y</td>
<td>0.48*Factor3, Errorvar. = 0.075</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>, R² = 0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>35.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.88</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Step Four: Testing Model Estimates

Since MLE as the optimal tool, the likelihood ratio chi-square ($\chi^2$), Wald statistics, and Lagrange multiplier an be used as the construct measurement model fit test. Other approaches of model fit index also provided by LISREL are the goodness of fit index, and the root mean square error. Whereas figure 6 and 7 show the much larger chi-square (model data fit not well), nevertheless our aim is to find the appropriate factor loadings for the variance and covariance decomposition. Table 10 shows the confirmatory construct factor loading for each index (observable) measure. The t statistics of the loadings and the $R^2$ of the construct measure exhibit high coefficient significance and good variance explanation of measure model.

Finally to computer the value at risk for the interest rate term structure of the SEM, we need to estimate the correlations of the three factor shown in table 11 by LISREL. With correlation of the factors and we can calibrate VaR figure as PCA and three factor-level, slope, and curvature have done. We can make

$$\Delta P/P = \Delta P/P|f1 + ... + \Delta P/P|f2$$

(25)

where $\Delta P/P$ is total percentage price change and $\Delta P/P|f_i$ is change in price due to factor. Then,

$$\sigma^2 (\Delta P/P) = \text{factor duration} \Omega \text{ factor duration'}$$

(26)

where $\Omega$ is the factor variance and covariance matrix and factor duration is the price change due to the factor change. In table 12, we show the factor durations for three factors: short, median, and long term factors according to our factor loadings and in table 13, we present the VaR estimates with the confirmatory factor SEM method.

Compared to table 6, the three factor SEM VaR measure has the approximate VaR to the level, slope, and curvature VaR measure since both of the factor models- SEM and level, slope, and curvature model measure part of the total variance ignoring the residual variances. Though, we
should note that the SEM three factors have the entirely different descriptions from the three factor level, slope, and curvature. The level, slope, and curvature uses all of the key rates as their factor construct, but the SEM use part of the key rates as their factor construct as seen in figure 6, figure 7, table 10, and table 5. While PCA components are independent of each other, SEM still owns some degree of correlations between its factors like level, slope, and curvature factor model.

The data model fit of the SEM also performs very poor. Hence it is not a good time series prediction. On the other hand, the level, slope, and curvature factor model can find its optimal fit of the model by the root mean square forecast errors and thus it is better tool for yield prediction. In addition, we see the high VaR sensitivity of the median term yield in SEM but the high VaR sensitivity of the curvature in level, slope, and curvature three factor model.

<table>
<thead>
<tr>
<th>TABLE 11 SEM CONFIRMATORY FACTOR CORRELATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) US Factor Corr.</td>
</tr>
<tr>
<td>Factor1</td>
</tr>
<tr>
<td>Factor1</td>
</tr>
<tr>
<td>Factor2</td>
</tr>
<tr>
<td>(-0.01)</td>
</tr>
<tr>
<td>Factor3</td>
</tr>
<tr>
<td>(-0.02)</td>
</tr>
<tr>
<td>note: In each factor block, covariance is in parenthesis and t statistics is at bottom.</td>
</tr>
</tbody>
</table>

| (b) Taiwan Factor Corr.                        |
| Factor1 | Factor2 | Factor3 |
| Factor1  | 1       |         |
| Factor2  | 0.99    | 1       |
| 0        | 1401.23 |         |
| Factor3  | 0.99    | 1       |
| 0        | 0       | 1       |
| 1422.63  | 4057.92 |         |
**TABLE 12 SEM CONFIRMATORY FACTOR (CASH FLOW) DURATIONS**

(a) US Bond Yield Factor Duration

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM3m</td>
<td>1.27</td>
<td>0</td>
<td>0</td>
<td>1.73</td>
</tr>
<tr>
<td>CM6M</td>
<td>1.24</td>
<td>0</td>
<td>0</td>
<td>3.19</td>
</tr>
<tr>
<td>CM1Y</td>
<td>1.24</td>
<td>0</td>
<td>0</td>
<td>17.91</td>
</tr>
<tr>
<td>CM2Y</td>
<td>0</td>
<td>1.14</td>
<td>0</td>
<td>31.85</td>
</tr>
<tr>
<td>CM3Y</td>
<td>0</td>
<td>1.02</td>
<td>0</td>
<td>19.75</td>
</tr>
<tr>
<td>CM5Y</td>
<td>0</td>
<td>0.81</td>
<td>0</td>
<td>5.79</td>
</tr>
<tr>
<td>CM7Y</td>
<td>0</td>
<td>0</td>
<td>0.66</td>
<td>4.28</td>
</tr>
<tr>
<td>CM10Y</td>
<td>0</td>
<td>0</td>
<td>0.48</td>
<td>1.68</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Factor Cash Flow: 28.236 71.113 5.654438

(b) Taiwan Bond Yield Factor Duration

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM3m</td>
<td>1.13</td>
<td>0</td>
<td>0</td>
<td>1.92</td>
</tr>
<tr>
<td>CM6M</td>
<td>1.26</td>
<td>0</td>
<td>0</td>
<td>3.55</td>
</tr>
<tr>
<td>CM1Y</td>
<td>0</td>
<td>1.33</td>
<td>0</td>
<td>19.90</td>
</tr>
<tr>
<td>CM2Y</td>
<td>0</td>
<td>1.34</td>
<td>0</td>
<td>35.39</td>
</tr>
<tr>
<td>CM3Y</td>
<td>0</td>
<td>0</td>
<td>1.34</td>
<td>21.95</td>
</tr>
<tr>
<td>CM5Y</td>
<td>0</td>
<td>0</td>
<td>0.17</td>
<td>10.86</td>
</tr>
<tr>
<td>CM7Y</td>
<td>0</td>
<td>0</td>
<td>1.33</td>
<td>6.43</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Factor Cash Flow: 6.64 73.88 39.80

**TABLE 13 VALUE AT RISK WITH SEM CONFIRMATORY FACTOR ESTIMATE**

(a) US Bond Yield VaR

<table>
<thead>
<tr>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
<th>CFA Cash Flow</th>
<th>VaR Delta</th>
<th>VaR Component</th>
<th>VaR Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor1 6.304E-08 1.077E-07 9.803E-08</td>
<td>28.236</td>
<td>0.00032</td>
<td>0.0090</td>
<td>10.48%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor2 1.077E-07 3.694E-07 3.807E-07</td>
<td>71.113</td>
<td>0.00100</td>
<td>0.0710</td>
<td>82.17%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor3 9.803E-08 3.807E-07 5.229E-07</td>
<td>5.654</td>
<td>0.00104</td>
<td>0.0059</td>
<td>6.86%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Var = 0.0858

(b) Taiwan Bond Yield VaR

<table>
<thead>
<tr>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
<th>CFA Cash Flow</th>
<th>VaR Delta</th>
<th>VaR Component</th>
<th>VaR Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor1 5.088E-08 3.814E-08 2.024E-08</td>
<td>6.65</td>
<td>0.00033</td>
<td>0.0022</td>
<td>6.71%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor2 3.814E-08 3.709E-08 1.935E-08</td>
<td>73.89</td>
<td>0.00031</td>
<td>0.0232</td>
<td>70.90%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor3 2.024E-08 1.935E-08 1.613E-08</td>
<td>39.81</td>
<td>0.00018</td>
<td>0.0073</td>
<td>22.39%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Var = 0.0327
CHAPTER V
CONCLUSION

Key rate duration application is the primary methodology used by JP Morgan for its bond portfolio value at risk measure. In reality, although there exist the catch issues of the real world key rate variance covariance structure and yield curve movement, many interest rate risk managers still prefer to use it for its simple and detail descriptions of real factor definitions. However, as to the interest rate risk researchers, the unstable key rate variance covariance and correlation structure and the unknown yield curve current movement often limit its usage for the long term interest rate risk management. Besides, the linear yield curve model is not suitable for the analysis of the nonlinear bond risk measure, e.g. the callable bond or the putable bond risk measures. At present, for the interest rate risk measure of the nonlinear derivative securities mentioned as above, we will refer to Wilson[1995] who presented Delta Gamma approximation. In the mean time, we refer to JP Morgan RiskMetrics who offered the Delta Gamma and variance-covariance matrix usages of the interest rate portfolio risk measure. In its recent interest rate risk documents, we can find the portfolio value at risk measure of bond option and foreign currency. The major risk measure process is to distribute the cash flow amount into delta, gamma, and theta cash flows and then combine variance covariance matrix of their cash flows to measure the portfolio bond value at risk.

The three factor- level, slope and curvature- interest rate model, in reality, might not fit to the general interest rate management. Nevertheless, since its distinctive characteristics among the three factors and its parsimonious features, this yield curve model should have some kind of advantages as well. Especially, while considering the economic changes in inflation, business cycle, and economic volatility, the three factors will explain significantly well.

We make use of the barbell and bullet bonds as an example to describe the three factor
effect upon the bond risk. When there is higher term structure level change, the higher cost barbell bond would have lower bond value at risk since its larger maturity term difference while when there is higher term structure slope change, the bullet bond would present lower bond value at risk and when there is higher term structure curvature change, the barbell bond would have lower bond value at risk since its larger bond convexity.

Principal component analysis of yield data is one kind of the parsimonious model. If only we analyze the specific component interest rates, we can forgo the component interest rate variance covariance and correlation structures. However, the explanation of the components is not easy for researchers and not to mention investors. Researchers tend to explain it as the level, slope, and curvature but it won't meet the factor independences that are contrast to the three factor model: level, slope, and curvature.

SEM is primarily used for the confirmatory factor analysis. Unlike the exploratory factor analysis trying to search the optimal factor loading while keeping the maximum variance of variable transformations, SEM will need the actual perception of the researchers in the filed of factor constructs (measurements) to find the appropriate measure and structure models. Though SEM is not fit the data model well (high chi-square and little p value), we would aim to find the factor loadings (variance decomposition) and thus covariance and correlation are the key input data for SEM. Compared to three factor model, SEM will do equally well for the VaR measure despite of its disappointing data model fit. Nevertheless, in reality, the researchers could construct good perceptible factors for investor risk management use as in our case: the short, median, and long term VaR measures and their VaR components as well as sensitivity analysis.
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